A BAYESIAN SIMULATION APPROACH TO MODELLING THE RELATIONSHIP BETWEEN RICE MILL LOSS RATE AND MOISTURE CONTENT WITH INCOMPLETE DATA: A CASE STUDY

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Abstract: The Classical Binary Logistic Regression model can be used in relating Rice Mill Loss Rate and Moisture content, but the shortcomings of this model in handling cases of incomplete or sparse data will always pose a challenge. This is because, to achieve the objective of eliminating seasonal effects, data need be aggregated over each month of the year in order to develop month - specific models. This has a negative effect of reducing sample size hence posing a challenge of incomplete data. In order to resolve problems of this nature, this work employed the Bayesian Simulation Modelling Approach in relating Rice Mill Loss Rate and Moisture content for each month of the year. This was done using the Markov Chain Monte Carlo (MCMC) algorithm implemented on the Windows Bayesian Inference Using Gibbs Sampling (WINBUGS) platform. Real time data on average moisture content (%), average number of rice bags (50 kg) processed and lost in each month of the year, were sourced from MIKAP Nigeria Limited, Makurdi, Nigeria and used in the study. Major results of the study shows that optimal moisture contents of spatio-temporal characteristic will reduce Rice Mill Loss Rate from the current 5% to 1-4% and that moisture content of rough rice is a risk factor to Rice Mill Loss Rate in the months of November to May.

Keywords: Bayesian, simulation, rice, loss rate

Introduction

In the last five decades, many researchers have pursued mathematical modelling of rice drying process with the key objective of developing a model for determining the moisture of the drying rice sample appropriate for optimal head rice yield during milling (Prakash and Pan, 2011). The Classical Binary Logistic Regression model can be used in relating Rice Mill Loss Rate and Moisture content, but the shortcomings of this model in handling cases of incomplete or sparse data will always pose a challenge (Peduzzi *et al*., 1996). Harvest moisture content of rice has been reported to have the greatest effect on rice milling quality and that harvesting at moisture content greater or less than optimal can cause a greater number of broken kernels (Jarrod and Terry, 2012). It is an obvious fact that most Rice Mill Factories mill rice at their convenience and not at the optimal moisture content at harvest. This is because they have stock piles of rough rice which they mill in any month of the year. This practice is typical of MIKAP Nigeria Limited, a Rice Mill located in Makurdi, Benue State, Nigeria were data was sourced for this research. The varieties of rice milled by this company are FARO 44 and FARO 52. The standard milling moisture content for these varieties is 13-14% (Dayo *et al.*, 2018). We argue as follows that this range of moisture content is neither factory location nor milling month specific. Hence it might not yield desired results.

The physical quality of milled rice has been shown to be affected by moisture content and relative humidity during delayed drying. In fact Moisture re-absorption of rough rice during delayed drying cause fissures and breakage of rice during milling (Tamrin *et al.*, 2017). Since relative humidity varies with the month of the year and from place to place (Agada *et al.*, 2017), then from the aforementioned, the moisture content of rice would also vary from month to month and from place to place. Modelling the relationship between Rice Mill Loss Rate and moisture content with the objective of eliminating seasonal effects would require aggregation of data for each month of the year and development of month - specific models. This has a negative effect of reducing sample size hence posing a challenge of incomplete data most especially when the Classical Binary Logistic Model is been used.

In order to resolve problems of this nature, this work employed the Bayesian Simulation Modelling Approach in relating Rice Mill Loss Rate and Moisture content for each month of the year. This was done using the Markov Chain Monte Carlo (MCMC) algorithm implemented on the Windows Bayesian Inference Using Gibbs Sampling (WINBUGS) platform. Some works that used the Bayesian Simulation Modelling Approach in handling incomplete data problems include those of Gemperi (2004), Koissi and Hogens (2005), Tripathi *et al.* (2019) and Agada *et al.* (2019). The rest of the paper is organized as follows; methodology, result & discussion, conclusion and recommendation

Materials and Methods

The binary logistic model relating monthly rice mill loss rate to moisture content of rice

We state the Binary Logistic Model as follows;

$$\text{Logit}\ (\pi_i) = \alpha_i + \beta_i\ (x_i - \bar{x}) \qquad (1)$$

Where: α_i and β_i are the logistic regression parameters, π_i (success rate) is the monthly Rice Mill Loss Rate. The covariate x_i is the monthly moisture content of rice (%) This was computed form data as the average of moisture contents at which rice was milled in a particular month.i =1, 2, 3, … 12 index the month of the year.

Also,

$$\text{Logit}\ (\pi_i) = \log\left(\frac{\pi_i}{1-\pi_i}\right) \qquad (2)$$

It follows that;

$$\pi_i = \frac{1}{1+e^{-[\alpha_i+\beta_i(x_i-\bar{x})]}} \qquad (3)$$

and

Centering the moisture content of rice (x_i) at the monthly mean ($\bar{x}$)

We centre the moisture content(x_i) of rice at the mean ($\bar{x}$). This mean moisture content was computed from data as the average of moisture contents over the twelve months of the year. Non-centering will mean zero moisture content which we consider unrealistic. Centering at the monthly mean moisture content will help determine the impact of a month's moisture content on the Rice Mill Loss Rate when it grows up to the average ($\bar{x}$) over the twelve months of the year.

Centering implies $x_i = \bar{x}$ and equation (3) becomes

$$\pi_i = \frac{1}{1+e^{-\alpha_i}} \quad (4)$$

If $\alpha_i < 0$ then;

$$\pi_i = \frac{1}{1+e^{\alpha_i}} \quad (5)$$

this has a negative impact on π_i

If $\alpha_i > 0$ then;

$$\pi_i = \frac{1}{1+e^{-\alpha_i}} \quad (6)$$

This has a positive impact on π_i

Effect of the monthly moisture content of rice (x_i) on the rice mill loss rate (π_i)

We establish mathematically, the effect of the monthly moisture content of rice x_i on the Rice Mill Loss Rate (π_i) of the i^th month. From equation (3)

$$\pi_i = \frac{1}{1+e^{-[\alpha_i+\beta_i(x_i-\bar{x})]}}$$

Observe that, if $\beta_i < 0$ then;

$\pi_i \to 0$ as $x_i$becomes large. This shows that increased and decreased levels of x_i has effect on decreasing and increasing π_i respectively. Hence, we term the monthly moisture content of rice (x_i) a Non-risk factor of Rice Mill Loss Rate (π_i).

Observe also from equation (3.5) that if $\beta_i > 0$ then;

$\pi_i \to 1$ as x_i becomes large. This shows that increased and decreased levels of x_i has the effect increasing and decreasing π_i respectively. Hence, we term the monthly moisture content of rice (x_i) a risk factor of Rice Mill Loss Rate (π_i).

We model the level of the risk (L_{Risk}) as;

$$L_{Risk} = \pi(\beta_i > 0) * 100\% \quad (7)$$

Where $P(\beta_i > 0)$ is the probability of having positive values of β_i. This is the proportion of time (%) that increased levels of monthly moisture content of rough rice has negative effect on the Rice Mill Loss Rate (π_i)

Determining the monthly moisture content of rice (x_i) for prescribed values of the rice mill loss rate (π_i)

The Rice Mill Loss Rate can be drastically reduced if we can determine the appropriate moisture content for prescribed values of loss rates.

From equation (3);

$$\pi_i = \frac{1}{1+e^{-[\alpha_i+\beta_i(x_i-\bar{x})]}}$$

it follows that

$$x_i = \bar{x} + {}^{1}\!/_{\beta_i}\left[\alpha_i + In\left(\frac{1-\pi_i}{\pi_i}\right)\right] \quad (8)$$

This equation relates the moisture content to prescribed values of Rice Mill Loss Rate(π_i) in this work.

The Bayesian binary logistic simulation model relating monthly rice mill loss rate to moisture content

As earlier mentioned, modeling the relationship between monthly Rice Mill Loss Rate with moisture content of rice as a covariate is not without the challenge of incomplete data. This is true when the Classical Binary Logistic Model is used. On the contrary, the Bayesian Binary Logistic Model does not bow to the challenge of incomplete data (Taeryon *et al.*, 2008). This is because unlike its classical counterpart which considers model parameters as fixed and data as random variables, it considers model parameters as random variables with known probability distributions and data as fixed. Hence it depends chiefly on model parameter sampling and not data sampling.

We therefore develop and implement a Bayesian Binary Logistic Simulation Modeling Procedure for modeling the relationship between monthly Rice MILL Loss Rate and Moisture Content of Rice. The modeling procedure embeds the Markov Chain Monte Carlo (MCMC) algorithm implemented on an Open Source Software Platform - Windows Bayesian Inference Using the Gibbs Sampler (WINBUG) (German & Geman, 1984).

The Bayesian statistical simulation modeling procedure

Given two faces of the coin; the Rice MILL Loss Rate (π_i) for a month i and the "no loss rate" $(1-\pi_i)$, we propose the Binomial Likelihood such that;

$y_i \setminus \pi_i \sim$ Binomial (π_i, n)

Where, y_i is the number of bags of rice lost in a month i while the success rate $\pi_i = {}^{y_i}\!/_{n_z}$ is Rice MILL Loss Rate in a month $i.n_z$ is the number of bags of rice supplied in a month. We state that the computation of π_i per $n(=1000)$ bags would be done in order to determine the observed values of y_i per 1000 bags and for computational ease.

Logistically, π_i is the transformation of the regression mean, $\alpha_i + \beta_i(x_i - \bar{x})$ and we state that;

$$Logit(\pi_i) = \alpha_i + \beta_i(x_i - \bar{x})$$

We suppose that the regression parameters α_i *and* β_i have the priors;

$\alpha_i \sim Normal(0, 0.01)$, $\beta_i \sim Normal(0, 0.01)$.

Using the parameter estimation version of the Bayes theorem (Scott, 2007), the posterior distributions of the model parameters α_i *and* β_i relating their respective prior densities ($f(\alpha_i)$ and $f(\beta_i)$) and their data likelihoods ($f(y_{i\setminus}\alpha_i)$ *and* $f(y_{i\setminus}\beta_i)$) are;

$$f(\alpha_i \setminus y_i) = \frac{f(\alpha_i)f(y_{i\setminus}\alpha_i)}{\int f(\alpha_i)f(y_{i\setminus}\alpha_i)d\alpha_i} \quad (9)$$

$$f(\beta_i \setminus y_i) = \frac{f(\beta_i)f(y_{i\setminus}\beta_i)}{\int f(\beta_i)f(y_{i\setminus}\beta_i)d\beta_i} \quad (10)$$

In WINBUG syntax, we shall fit the Bayesian Logistic Regression Model with centered covariate as follows;

```
Model {
for (i in 1: 12){
y[i] ~ dbin(p[i],1000)
logit(p[i])<- alpha[i] + beta[i]*(x[i]-mean(x[]))
MC4[i] <- ((logit(0.996)-alpha[i])/beta[i]) + mean(x[])
MC3[i] <- ((logit(0.997)-alpha[i])/beta[i]) + mean(x[])
MC2[i] <- ((logit(0.998)-alpha[i])/beta[i]) + mean(x[])
MC1[i] <- ((logit(0.999)-alpha[i])/beta[i]) + mean(x[])
prob[i] <- step(beta[i] - 0.5)
alpha[i] ~ dnorm(0,0.01)
beta[i] ~  dnorm(0,0.01)}
}
```

Where the data list of bags lost $y[\ \ ]$ and monthly moisture content of rice (in percentage) $x[\ \ ]$ as well as the initialization list for the model parameter arrays; $alpha[\ \ ]$ *and* $beta[\ \ ]$are defined for each month i. k is used to index the months of the year of interest while n is set at 1000. The simulation was run for 100,000 burn-ins after which samples were collected for 100,000 iterations. A thinning of 32 would be maintained throughout the simulations and the overlay check box in WINBUG checked to reduce autocorrelation. Other modeling requirements are as stated by the WINBUG Software documentation.

We mention that WINBUG uses the equation;

$$\pi = \frac{1}{1+e^{-(alpha-beta(x-\bar{x})}} \quad (11)$$

in computing the Rice Mill Loss Rate (π_i) for a month i. This gives the simulated value of π_i. The prescribed values of Rice Mill Loss Rate are 1, 2, 3 and4 % while the current loss rate is 5%

Model convergence diagnostic check

Model convergence diagnostics was done using history plots, density plots and autocorrelation plots. The plots were produced when the model parameters and measures were monitored on WINBUG. Our approach for investigating convergence issues is by inspecting the mixing and time trends within the chains of individual parameters. The history plots are the most accessible convergence diagnostics and are easy to inspect visually. It plots the simulated values for the parameter against the iteration number. The history plot of a well-mixing parameter should traverse the posterior domain rapidly and should have nearly constant mean and variance.

The density plots of the model parameters were checked against their actual probability distributions to see whether the right distribution is simulated. This was done for the alpha and beta distribution for each month$i$.

Samples simulated using MCMC methods are correlated. The smaller the correlation, the more efficient the sampling process. Though, the Gibbs - MCMC algorithm typically generates less-correlated draws, there is a need to monitor the autocorrelation of each parameter to ensure samples are independent. The autocorrelation plot that comes from a well-mixing chain becomes negligible fairly quickly, after a few lags. This was achieved for the model parameters and measures.

Results and Discussion

The study results include actual dataset on average moisture content (%) and actual average number of rice bags processed and lost in each month of the year. The results of the Bayesian Statistical Simulation which include; the simulation model parameter and measure values for each month of the year and the model diagnostic checks results were also presented. The results of the model diagnostic checks include history plots, density and autocorrelation plots of the model parameters alpha and beta. Further results of the study include a distribution of month – specific moisture content risk factor status, level of risk and impact on Rice Mill Loss Rate. In addition, the simulated moisture content for 1-4% prescribed values of Rice Mill Loss Rate was also presented.

Table 1 presents actual dataset on average moisture content (%) and actual average number of 50 kg bags processed and lost in each month of the year. The actual Rice Mill Loss Rate of 5% was estimated from the data on average number of rice processed and lost in each month of the year. These values were computed from actual aggregate data for the year 2017. The number of rice bags lost per 1000 processed bags were determined for each month of the year and used as number of trials for the Binomial distribution in the course of the Bayesian statistical modelling. The moisture content for each month of the year also serves as input to the model.

Table 1: Distribution of average moisture content and average number of rice bags processed and lost

Month	Moisture content (%)	Number of 50 kg bags processed	Number of 50 kg bags lost
January	11.20	6300800	31504
February	11.10	3332800	16664
March	11.50	4332000	21660
April	11.30	5475600	27378
May	11.00	5907800	29539
June	13.10	8590600	42953
July	12.20	8083000	40415
August	12.40	8097400	40487
September	13.40	9589000	47945
October	13.40	8475800	42379
November	11.00	6050800	30254
December	11.80	1992800	9964

As earlier mentioned, the incompleteness of this data (aggregates) limits the use of the Classical Logistic Regression Model (Taeryon *et al*., 2008). This limitation calls for the development of a Bayesian Statistical Modelling Procedure using the MCMC – Gibbs algorithm on the WINBUG platform.

After the development of the model, convergence diagnostic checks were conducted for each model parameter in order to ascertain model adequacy. The history plot, density plots and autocorrelation plots were used for this purpose. Though, all these plots were made for each model parameter alpha and beta, sample plots were presented in Figs. 1 – 4. Observe that the history plots shows that the model parameters are well – mixed. This is because they traverse the posterior domain rapidly with nearly constant mean and variance. The model prior distribution for alpha and bêta is normal (0, 0.01). The density plots of these priors reflect this distribution which further validates the model. The autocorrelation plots of each parameter and measure depict the independence of the samples generated. This is because the autocorrelations become negligible fairly quickly, after a few lags.

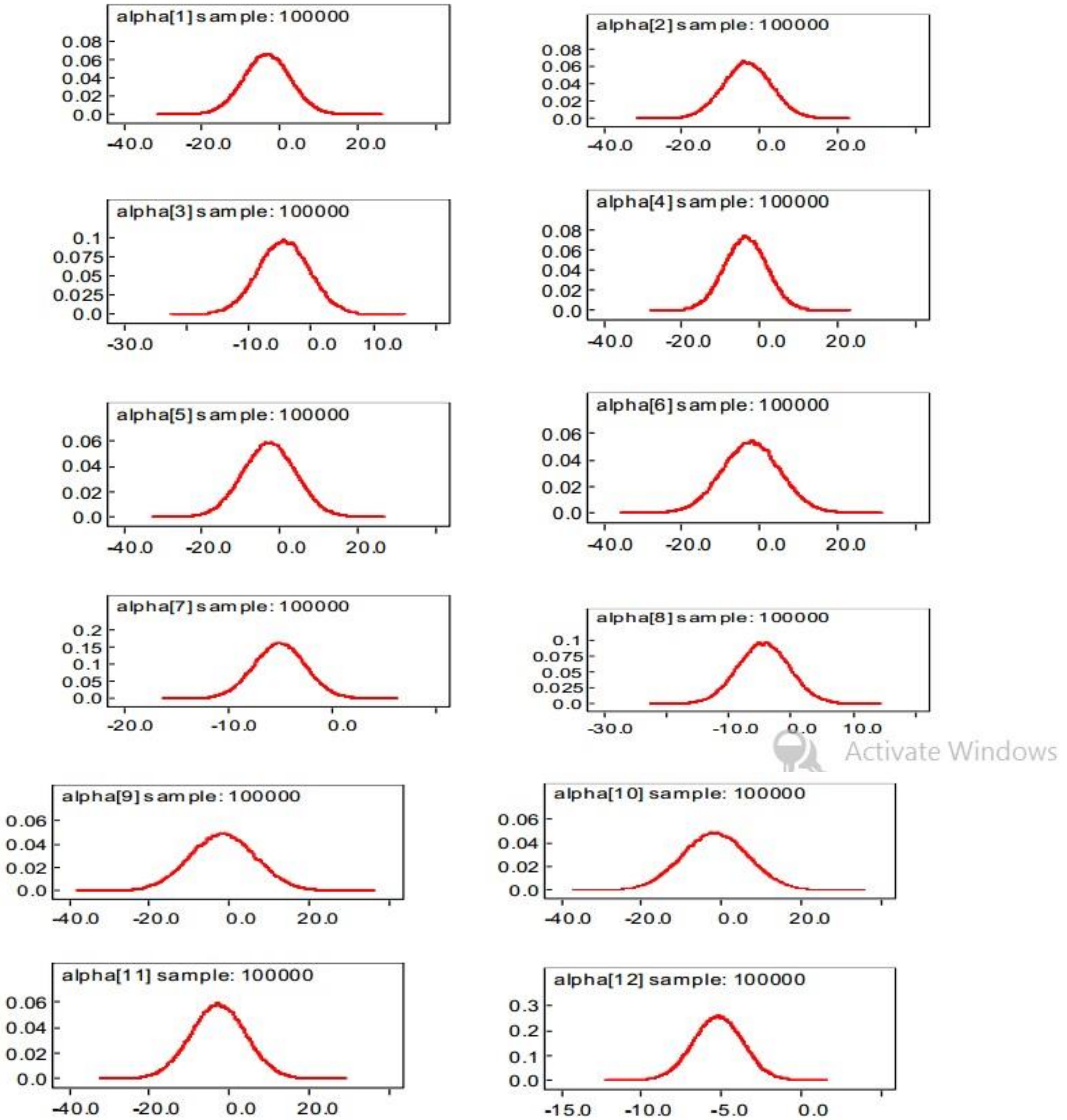
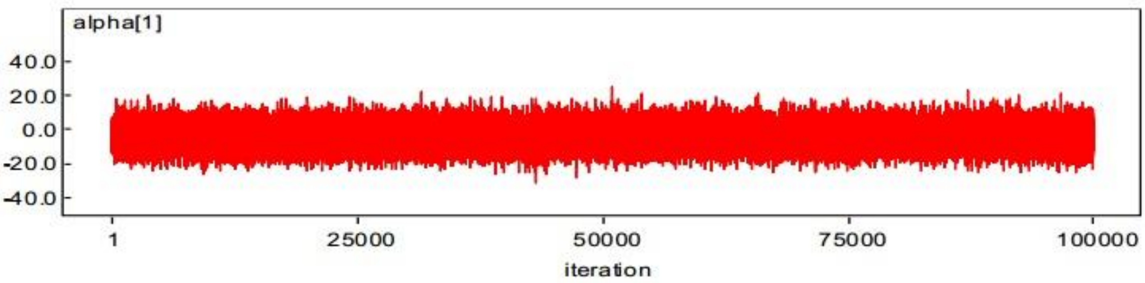


Fig. 1: Density plots of model parameter alpha for January to December



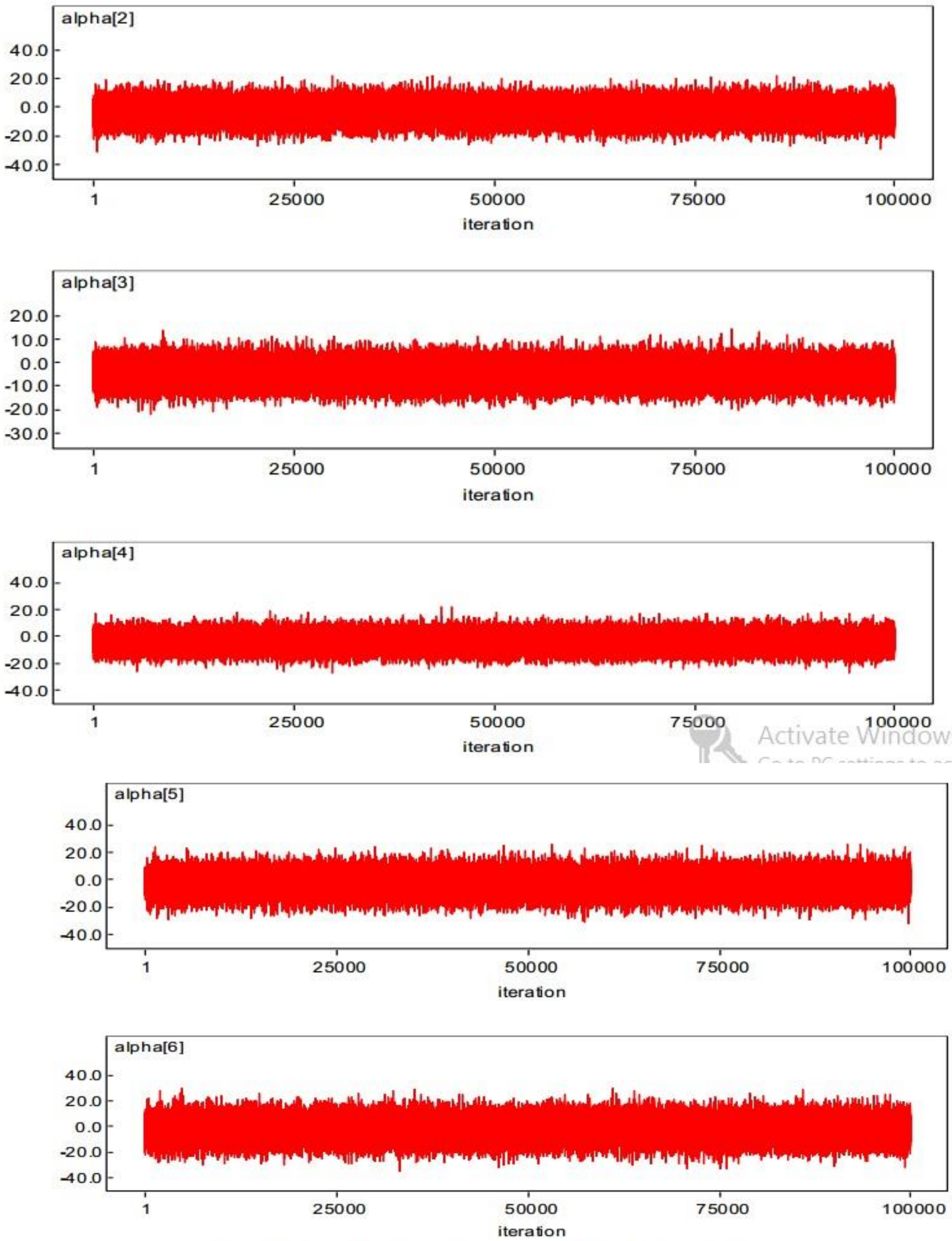
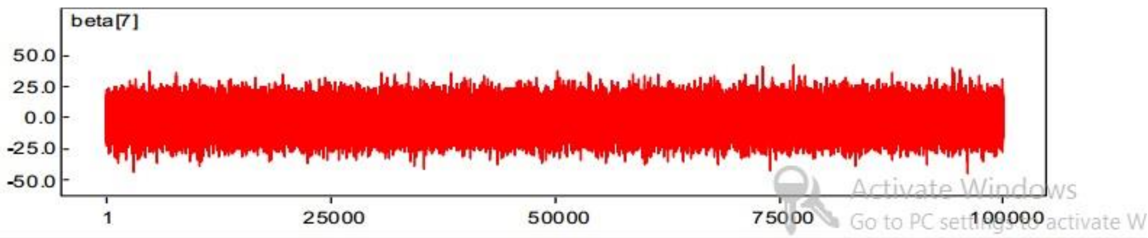


Fig. 2: History plots of model parameter alpha for January to June



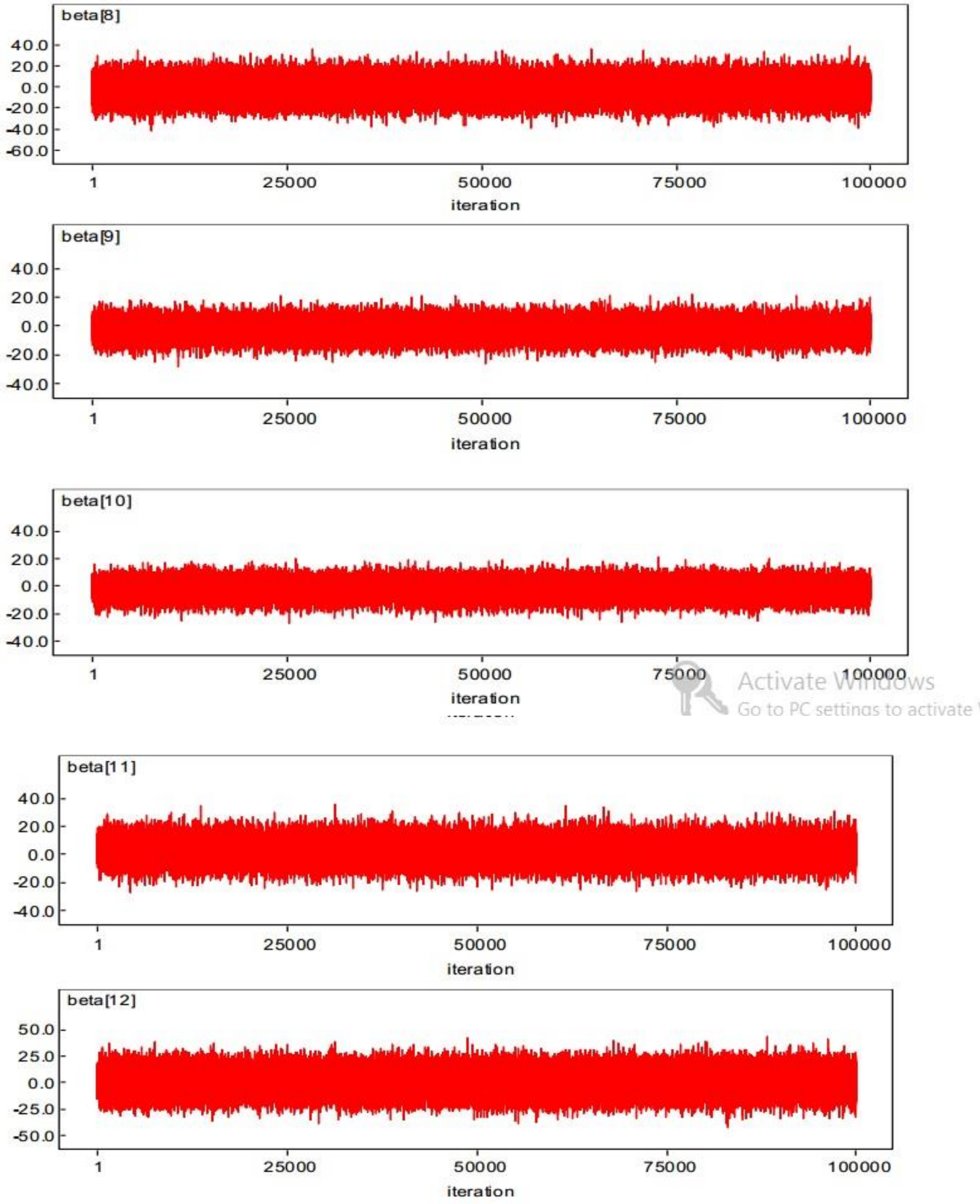


Fig. 3: History plots of model parameter beta for July to December

Fig. 3: History plots of model parameter beta for July to December

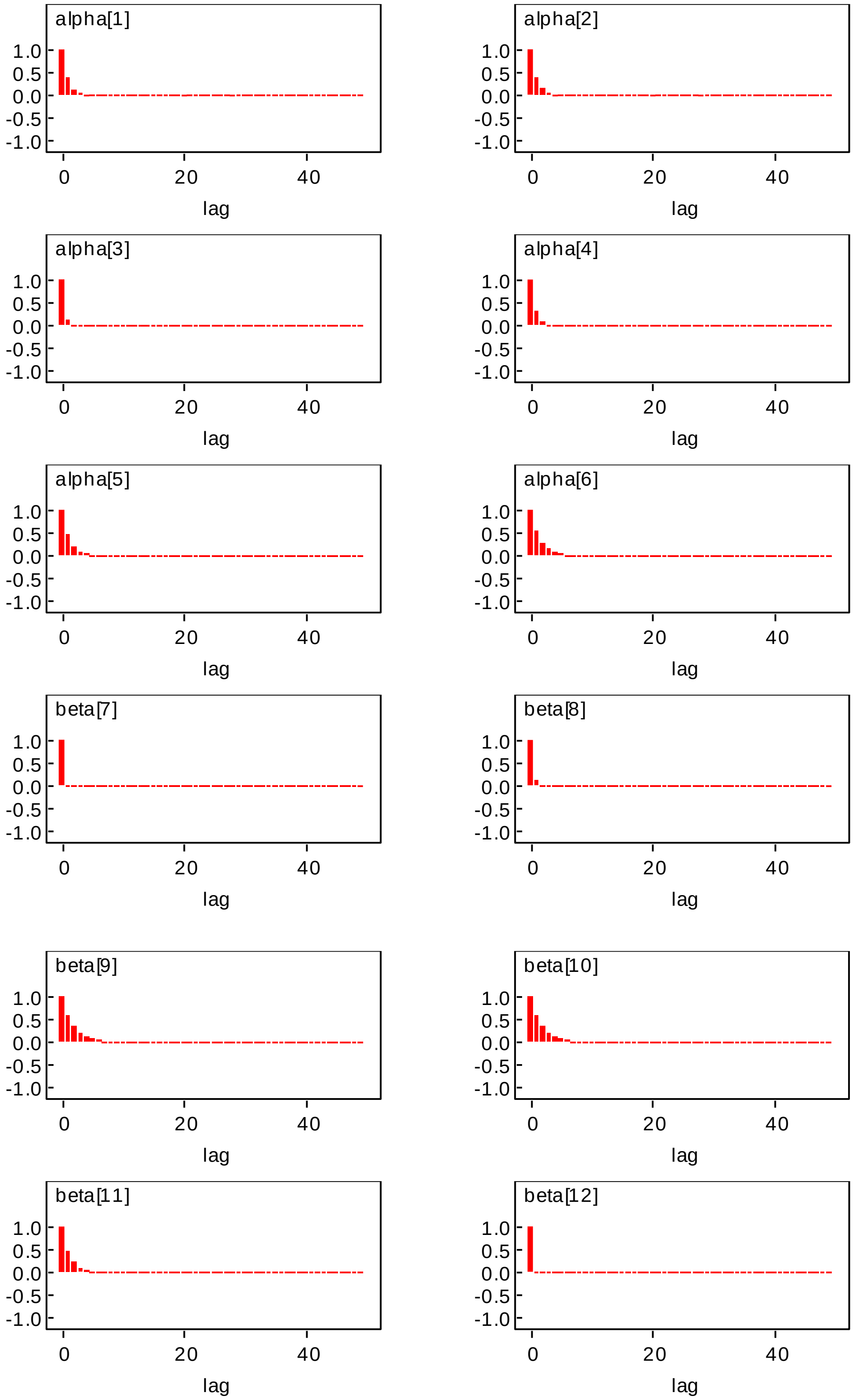


Fig. 4: Autocorrelation plots of model parameters alpha (January – June) and beta (July to December)

Model parameters and measure values for each month of the year are captured on Table 2. Details on these tables include their mean, standard deviation, Monte Carlo Simulation Error, median and the 95% credible interval. The model parameters are alpha and beta as earlier mentioned while the measures are the moisture content impact on Rice Mill Loss Rate, it's risk factor status and level of risk (prob (beta > 0)). The values of alpha and beta for each month the year helps to determine the relationship between the Rice Mill Loss Rateand moisture content for that month. This is achieved when they are plugged into equation (1).

Table 2: Model parameters and measure values for each month of the year

Node	Mean	SD	MC error	2.5%	Median	97.5%
MC1[1]	33.98	6522.0	20.56	-11.84	12.25	34.5
MC1[2]	10.15	785.5	2.475	-12.23	12.24	35.39
MC1[3]	11.43	421.5	1.335	-9.561	12.36	32.76
MC1[4]	13.46	496.9	1.569	-11.44	12.29	34.03
MC1[5]	12.07	408.8	1.315	-13.99	12.2	37.32
MC1[6]	12.04	650.0	2.063	-13.82	11.77	40.16
MC1[7]	19.7	2332.0	7.361	-7.987	11.48	32.08
MC1[8]	12.3	309.1	0.9791	-8.048	11.56	33.16
MC1[9]	11.84	610.1	1.941	-18.64	11.85	45.26
MC1[10]	16.73	855.0	2.698	-17.55	11.86	45.31
MC1[11]	13.59	544.6	1.729	-13.23	12.19	36.87
MC1[12]	12.05	1493.0	4.724	-8.731	12.45	30.97
MC2[1]	32.75	6172.0	19.46	-10.54	12.19	33.18
MC2[2]	10.21	740.3	2.333	-10.89	12.18	34.0
MC2[3]	11.44	397.5	1.259	-8.374	12.31	31.55
MC2[4]	13.34	469.6	1.483	-10.13	12.23	32.77
MC2[5]	12.01	386.3	1.242	-12.55	12.13	35.88
MC2[6]	12.1	613.2	1.946	-12.33	11.85	38.65
MC2[7]	19.28	2202.0	6.949	-6.868	11.53	30.95
MC2[8]	12.3	291.9	0.9245	-6.882	11.61	31.99
MC2[9]	11.92	574.9	1.83	-16.87	11.94	43.46
MC2[10]	16.53	804.5	2.539	-15.81	11.94	43.54
MC2[11]	13.45	514.5	1.634	-11.87	12.12	35.41
MC2[12]	12.04	1410.0	4.461	-7.54	12.41	29.86
MC3[1]	32.03	5968.0	18.81	-9.783	12.15	32.41
MC3[2]	10.25	713.8	2.249	-10.1	12.14	33.19
MC3[3]	11.45	383.4	1.214	-7.681	12.28	30.82
MC3[4]	13.27	453.6	1.433	-9.394	12.2	32.02
MC3[5]	11.97	373.1	1.2	-11.73	12.09	34.99
MC3[6]	12.13	591.7	1.877	-11.42	11.89	37.77
MC3[7]	19.04	2126.0	6.708	-6.188	11.55	30.3
MC3[8]	12.31	281.8	0.8925	-6.218	11.64	31.3
MC3[9]	11.97	554.3	1.764	-15.77	11.99	42.43
MC3[10]	16.41	775.0	2.446	-14.8	11.99	42.48
MC3[11]	13.36	496.8	1.578	-11.08	12.08	34.56
MC3[12]	12.03	1361.0	4.307	-6.877	12.39	29.2
MC4[1]	31.52	5822.0	18.35	-9.241	12.13	31.88
MC4[2]	10.28	695.0	2.19	-9.557	12.12	32.62
MC4[3]	11.45	373.4	1.182	-7.197	12.26	30.34
MC4[4]	13.22	442.3	1.397	-8.866	12.17	31.47
MC4[5]	11.95	363.7	1.17	-11.11	12.07	34.36
MC4[6]	12.16	576.4	1.829	-10.81	11.92	37.14
MC4[7]	18.87	2071.0	6.537	-5.717	11.57	29.83
MC4[8]	12.31	274.6	0.8697	-5.739	11.66	30.81
MC4[9]	12.01	539.7	1.718	-15.01	12.02	41.68
MC4[10]	16.33	754.0	2.379	-14.04	12.03	41.74
MC4[11]	13.3	484.3	1.538	-10.5	12.05	33.96
MC4[12]	12.03	1327.0	4.198	-6.385	12.38	28.73
alpha[1]	-3.461	6.073	0.02883	-15.35	-3.434	8.385
alpha[2]	-3.394	6.15	0.0286	-15.43	-3.407	8.609
alpha[3]	-4.468	4.171	0.01514	-12.65	-4.471	3.691
alpha[4]	-3.755	5.499	0.02248	-14.49	-3.739	7.002
alpha[5]	-2.81	6.881	0.03625	-16.41	-2.788	10.59
alpha[6]	-2.332	7.532	0.04063	-17.09	-2.342	12.52
alpha[7]	-5.092	2.398	0.007989	-9.816	-5.086	-0.399
alpha[8]	-4.517	4.073	0.01391	-12.49	-4.519	3.488
alpha[9]	-1.787	8.225	0.05481	-17.93	-1.783	14.36
alpha[10]	-1.694	8.242	0.04973	-17.76	-1.736	14.54
alpha[11]	-2.879	6.906	0.03619	-16.38	-2.894	10.69
alpha[12]	-5.245	1.608	0.00485	-8.389	-5.239	-2.082
beta[1]	2.549	8.018	0.03801	-13.15	2.58	18.21
beta[2]	2.566	7.911	0.03671	-12.92	2.542	18.01
beta[3]	2.009	9.098	0.03281	-15.82	1.988	19.79
beta[4]	2.485	8.358	0.03414	-13.84	2.506	18.84
beta[5]	2.695	7.192	0.03802	-11.5	2.728	16.7
beta[6]	-2.675	6.59	0.0355	-15.67	-2.668	10.26
beta[7]	-1.205	9.686	0.03186	-20.12	-1.203	17.8
beta[8]	-1.955	9.148	0.03115	-19.95	-1.951	15.96
beta[9]	-2.498	5.701	0.03806	-13.66	-2.495	8.684
beta[10]	-2.561	5.712	0.03436	-13.8	-2.551	8.582
beta[11]	2.624	7.218	0.0379	-11.47	2.618	16.76
beta[12]	0.8867	9.858	0.03004	-18.37	0.875	20.36
p[1]	0.005024	0.002224	7.008E-6	0.00165	0.0047	0.01023
p[2]	0.005039	0.002243	6.911E-6	0.001664	0.0047	0.01029
p[3]	0.005052	0.002245	7.053E-6	0.001652	0.0047	0.0103
p[4]	0.005039	0.002236	6.507E-6	0.001654	0.0047	0.01024
p[5]	0.005036	0.002235	7.032E-6	0.001637	0.00472	0.01025
p[6]	0.005024	0.002231	6.846E-6	0.001641	0.00469	0.01024
p[7]	0.005047	0.002235	7.35E-6	0.001663	0.00472	0.01028
p[8]	0.005057	0.002249	7.636E-6	0.001666	0.00472	0.01033
p[9]	0.005018	0.00224	6.84E-6	0.001635	0.00469	0.01027
p[10]	0.005027	0.002234	7.074E-6	0.001644	0.00476	0.01026
p[11]	0.005034	0.002238	7.011E-6	0.001641	0.00470	0.01025
p[12]	0.005056	0.002245	6.914E-6	0.001661	0.00473	0.01031
prob[1]	0.6009	0.4897	0.00207	0.0	1.0	1.0
prob[2]	0.6034	0.4892	0.002023	0.0	1.0	1.0
prob[3]	0.5657	0.4957	0.001673	0.0	1.0	1.0
prob[4]	0.5933	0.4912	0.001895	0.0	1.0	1.0
prob[5]	0.6202	0.4853	0.002186	0.0	1.0	1.0
prob[6]	0.3157	0.4648	0.002113	0.0	0.0	1.0
prob[7]	0.43	0.4951	0.001581	0.0	0.0	1.0
prob[8]	0.3949	0.4888	0.001681	0.0	0.0	1.0
prob[9]	0.2972	0.457	0.002573	0.0	0.0	1.0
prob[10]	0.2955	0.4563	0.002224	0.0	0.0	1.0
prob[11]	0.6157	0.4864	0.002195	0.0	1.0	1.0
prob[12]	0.5152	0.4998	0.001551	0.0	1.0	1.0

Note (i) The numbers 1 – 12 index the month of the year
(ii) model parameters are alpha and beta
(iii) model measures are rice mill loss rate (p), risk magnitude of moisture content (prob)
(iv) MC1, MC2, MC3 and MC4 denote moisture contents for 1, 2, 3 and 4 % prescribed values of Rice Mill Loss Rates

Table 3: Moisture contents at 5% current rice mill loss rate and 1-4% simulated rice mill loss rates

	Current Rice Mill Loss Rate	Simulated Rice Mill Loss Rate			
Month	**5 %**	**4 %**	**3 %**	**2 %**	**1 %**
January	11.20	12.13	12.15	12.19	12.25
February	11.18	12.12	12.14	12.18	12.24
March	11.50	12.26	12.28	12.31	12.36
April	11.30	12.17	12.20	12.23	12.29
May	11.00	12.07	12.09	12.13	12.30
June	13.10	11.92	11.89	11.85	11.77
July	12.20	11.57	11.55	11.53	11.48
August	12.40	11.66	11.64	11.61	11.56
September	13.40	12.02	11.99	11.94	11.85
October	13.40	12.03	11.99	11.94	11.86
November	11.00	12.05	12.08	12.12	12.19
December	11.80	12.38	12.39	12.41	12.45

Note: moisture contents are median values

As established in the mathematical analysis, the sign of the model parameter alpha assists in determining the impact (positive or negative) of a month's moisture content on the Rice Mill Loss Rate of that month. This was ascertained in this study when a month's moisture content equals the average (or expected value)over the twelve months of the year.The sign of beta assists in determining whether the moisture content of the month in question is a risk factor of the Rice Mill Loss Rate or not. The level of this risk is computed using equation (6). As earlier mentioned, it is the proportion of time (%) that increased (reduced) levels of moisture content of a particular month, has positive (negative) effect on the Rice Mill Loss Rate of that month. The overall results shows that if each months moisture content equal the average over the twelve months of the year, moisture content of rice will negatively impact on the Rice Mill Loss Rate see Table 4. In other words, it reduces the actual or observed Rice Mill Loss Rate of each month. It can be inferred that if emphasis is not on achieving specific rice mill loss rates, then maintaining a month's moisture content at average value over the months of the year may suffice in reducing loss rate. Furthermore, moisture content is a risk factor to Rice Mill Loss Rate in the months of November – May (dry months), while it is a non-risk factor in the months of June – October (wet months). This shows that moisture content has seasonal effect on Rice Mill Loss Rate. Details of these results are on Table 4. Still on Table 4, Moisture Content Risk Levels are higher in the dry months with a range of 51.52 – 62.02 % and lower in the wet months with a range of 29.55 – 43.00 %. Hence there is a high chance of rough rice moisture re-absorption in the wet months and lose in the dry months. Thus it becomes apparent that, there is a need to determine Month – Specific Optimal Rice Mill Loss Rates local to MIKAP Nigeria Limited due to the spatio-temporal characteristic of moisture content. Recall that values in these ranges are the proportion of times (%) that increased (reduced) levels of moisture content have positive (negative) effect on Rice Mill Loss Rate.

Table 4: Distribution of month specific moisture content risk factor status, level of risk and impact on Rice Mill Loss Rate

Month	Sign of model parameter (α)	Impact of moisture content on RMLR when $x_i = \bar{x}$	Sign of model parameter (β)	Moisture content factor status	Moisture content level of risk (%)
January	-	Negative	+	Risk factor	60.10
February	-	Negative	+	Risk factor	60.34
March	-	Negative	+	Risk factor	56.57
April	-	Negative	+	Risk factor	59.33
May	-	Negative	+	Risk factor	62.02
June	-	Negative	-	Non-risk factor	31.57
July	-	Negative	-	Non - Risk factor	43.00
August	-	Negative	-	Non - Risk factor	39.49
September	-	Negative	-	Non - risk factor	29.72
October	-	Negative	-	Non - Risk factor	29.55
November	-	Negative	+	Risk factor	61.57
December	-	Negative	+	Risk factor	51.52

Note : (i) RMLR = Rice Mill Loss Rate

(ii)$x_i = \bar{x}$implies monthly moisture content equals it's average over the twelve months

Actual moisture content at 5% Rice Mill Loss Rate and Simulated values of moisture content at 1, 2, 3 and 4 % prescribed values of Rice Mill Loss Rates are captured in Table 3. Observe on this table that simulated median values of moisture contents are used. This is because of the high variability in the values. See mean and standard deviations on Table 2. We envisage that the median value would better depict the centrality of the data distribution under this circumstance.

In the dry months of the year, observe that the moisture Contents at the current Rice Mill Loss Rate of 5 % are lower than those of the Simulated Rice Mill Loss Rates of 1 – 4 %. In the wet months, they are higher. See table 3 for details. This is perhaps the reason for MIKAP's higher loss rate of 5%. Recall that rough rice re-absorbs or loses moisture in a particular month of the year which indeed causes fissures and breakage of rice during milling (Tamrin *et al.*, 2017). The fact that MIKAP Nigeria Limited store stock piles of rough rice over the year and mill at their convenience in any month of the year, moisture re-absorption or loss in the month in question will not allow for optimal moisture content that will minimize the Rice Mill Loss Rate. Nevertheless, haven determined Month-Specific moisture contents that will reduce Rice Mill Loss Rate to 1 – 4% in this study, the company may wish to seek scientific methods of maintaining moisture content at these optimal values for the loss rate so desired.

We also mention that the standard moisture content of 13 – 14% for FARO 44 and FARO 52 earlier stated in this work is neither Rice Mill Factory Location nor milling month specific. This is because moisture content varies with geographical location and time as earlier established. Hence localized values of simulated moisture contents determined in this work for prescribed values of Rice Mill Loss Rates, will better the lots of MIKAP Nigeria Limited.

Conclusion and Recommendations

The following conclusions were drawn from the study;

(i) A Bayesian Binary Logistic modelling procedure was developed for relating Rice Mill Loss Rate and Moisture Content using incomplete data.

(ii) Milling rice at the average value of moisture content over the twelve months of the year will negatively impact on the Rice Mill Loss Rate of MIKAP Nigeria Limited.

(iii) Moisture content is a risk factor toRice Mill Loss Rate in the months of November – May (dry months), while it is a non-risk factor in the months of June – October (wet months).

(iv) Optimal moisture contents of spatio-temporal characteristic that will reduce Rice Mill Loss Rate to 1 – 4 % have been determined in this study

The study recommends that;

(i) This modelling procedure should be applied to similar modelling problems with the challenge of incomplete data.

(ii) If emphasis is not on achieving specific rice mill loss rates, then it suffices for MIKAP Nigeria Limited to mill rice at a value of moisture content that equals the average over the months of the year.

(iii) MIKAP Nigeria Limited should be careful when milling rice in the dry months of the year (November – May) since moisture content is a risk factor to Rice Mill Loss Rate in these months .

(iv) MIKAP Nigeria Limited may wish to seek scientific methods of maintaining the Month-Specific moisture contents that will reduce Rice Mill Loss Rate to the 1 – 4 % prescribed values.

Conflict of Interest

The authors declare that there is no conflict of interest reported in this work.

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